Description of Ancillary Files for "The electric and magnetic form factors of the proton"

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1 Form factors fits

1.1 To Mainz data alone

The ASCII files in *fits/MainzOnly* contain the extracted form factors and uncertainties from fits of the different models to the data of this experiment alone. The used cross sections include the Feshbach Coulomb-Correction. The file format is as follows:

- Each row is one record.
- All errors are absolute values representing the 68% point-wise confidence band.
- Column 1: Q^2 in GeV^2.
- Column 2: G_E.
- Column 3: The statistical uncertainty of G_E.
- Column 4+5: The experimental systematic uncertainty. Column 4 is the difference to the upper bound, column 5 to the lower, i.e., the range goes from column 2 - column 5 to column 2 + column 4.
- Column 6+7: The result when varying the applied Coulomb-correction by $\pm 50\%$. Column 6 is difference to upper bound, column 7 to the lower.
- Column 8-13: The equivalent of 2-7 for G_M/μ_p.
- Column 14-19: The equivalent of 2-7 for $\mu_p G_E/G_M$.

1.2 With the inclusion of world data

The files in *fits/World/CrossSectionsOnly* contain fits to the Mainz data and external cross section data. The cross section data include the Feshbach Coulomb-Correction.

The files in *fits/World/CrossSectionAndPolarized* contain fits to the Mainz data and data from external cross section and polarized ratio measurements. The fit include the phenomenological TPE model, on top of the Feshbach Coulomb-Correction, as described in the main paper.

The data format is as follows:

- Each row is one record.
- All errors are absolute values representing the 68% point-wise confidence band.
- Column 1: Q^2 in GeV^2.
- Column 2: G_E.
- Column 3: The statistical uncertainty of G_E.
- Column 4+5: For the spline fit: The model dependency. Column 4 is the difference to the upper bound, column 5 to the lower, i.e., the range goes from column 2 - column 5 to column 2 + column 4. For the Pad'e-Fit, these columns are 0.
- Column 6-9: The equivalent of 2-5 for G_M/μ_p.
- Column 10-13: The equivalent of 2-5 for $\mu_p G_E/G_M$.
- Column 14-17: (For the second data set only) The equivalent of 2-5 for the value of $A = -a \ln(b\,Q^2 + 1)$, i.e., the additional TPE correction is $A\,(1 - \varepsilon)$.

2 Rosenbluth extraction

The file *Rosenbluth/Rosenbluth.dat* contains the form factors extracted using the Rosenbluth-Method as described in the main paper. The data format is as follows:

- Each row is one record.
- All errors are absolute values.
- Column 1: Q^2 in GeV^2.
- Column 2: G_E.
- Column 3: The statistical uncertainty of G_E.
- Column 4: G_M.
- Column 5: The statistical uncertainty of G_M.

3 Cross sections

The file *CrossSections/CrossSections.dat* contains the cross section data of this experiment. The data format is as follows:

- Each row is one record
- Column 1: Beam energy, in MeV.
- Column 2: Used spectrometer: the letter A,B or C.
- Column 3: Central Angle of spectrometer, in degree. This is the center of the in-plane acceptance. Do not use for calculations of kinematic variables.
- Column 4: The acceptance-averaged Q^2 in GeV^2. Kinematic variables should be calculated from this and the energy.
- Column 5: The experimental cross section divided by the simulated cross section using the standard dipole. The normalization is taken from the spline fit.
- Column 6: Point-to-Point uncertainty of column 5, i.e., the scaled errors from counting statistics.
- Column 7/8: The experimental cross section when the cut in the energy loss is varied to be smaller/larger. Normalization is fit anew.
- Column 9: The applied Coulomb-correction. To get uncorrected values, one has to multiply columns 5-8 by column 9.
- Column 10: The factor applied to calculate the experimental systematic error bounds. See next section.
- Column 11: A string representing which normalization parameters govern this measurement. See next section.
- Columns 12-18: The relative normalization using the other fit models. One has to multiply columns 5 and 6 with one of these numbers to get the cross section with normalization from that model. The column order is 12: Spline times dipole, 13: Polynomial, 14: Polynomial times dipole, 15: Polynomial plus dipole, 16: Inverse polynomial, 17: Friedrich-Walcher, 18: Double dipole.

The point-to-point uncertainties in column 6 are calculated from the statistical uncertainties by scaling. The corresponding scale factors are determined independently for each spectrometer and energy combination. Thery are given in Table 1.

Spectrometer/Energy	A	B	C
180 MeV	1.070	1.266	1.366
315 MeV	1.753	2.080	2.293
450 MeV	1.654	1.550	1.833
585 MeV	1.215	1.081	1.766
720 MeV	1.465	1.700	1.100
855 MeV	1.100	1.700	1.170

Table 1: Table of the scaling factors for the point-to-point errors.

4 How to use the data for comparisons/fits

To compare a calculation of the form factors or a fit to the results of this paper, it's easiest to compare to the extracted form factors from. Comparing two functions, one has to take care in the interpretation of the uncertainties.

For calculations which need the form factor as input, it may be best to interpolate between the tabulated values. While closed form expressions are possible for the form factor values, the uncertainties are not expressed easily with a functional form.

A fit can be performed using three approaches:

1. Using the tabulated form factor values. Since the values are extracted using a certain model, the fit result may be very biased and it is hard to produce correct uncertainty estimates. However, it's the easiest way to test the overall behavior of a fit function without setting up a more elaborate fit.

2. Fit to the cross sections without floating normalization. This fit will still be biased, since the normalization is taken from a model, however, flexible models are shown to produce very similar normalizations. However, a treatment of the systematic errors is difficult.

3. Fit to the cross section, with floating normalization. This is the optimal method and produces an unbiased result.

For the last approach, the fit has to include the normalization parameters in addition to the model parameters. A fit has typically an inner loop where the model is evaluated with the current set of parameter values. The result is then compared to the data, e.g., for a χ^2 fit, the difference between data and model is divided by the error estimate and the result is squared. The sum is then minimized by the fit algorithm. To handle the normalization parameters, one first has to build the product of the normalization parameters given in column 11, e.g., 1:3 would indicate the product of parameter 1 and 3. The model prediction is then multiplied by this value. Alternatively, one could divide both data and uncertainty by this value. The fit result for the model parameters is independent from the normalization already applied to the data, however the

normalization parameters have to be understood relative to the normalization already included.

To gauge systematic effects, in this work, we multiply the cross section values with an estimate of possible angle dependent systematics. For convenience, the factor is given in column 10. To produce the corresponding uncertainty bands, one has to multiply the values in column 5 and 6 by column 10 and perform a new fit (with new normalization). The difference to the fit without the modification gives one side of the confidence band, a repetition with division instead of multiplication gives the other.

For the calculation of kinematic variables, it is important to use the average Q^2 and the energy and to not rely on the given central scattering angle.

5 Example fitting program

As a template, a minimal fitting program, written in python, is provided as *fitexample.py*. It will fit a polynomial fit to the data with floating normalization. It expects two arguments: the filename of the cross sections (in the format described above) and an output filename. When called, it will read in the data, fit the model and save the calculated form factors values to the specified file, overwritting the contents if the file already exists.