Ultraperipheral collisions at the Large Hadron Collider

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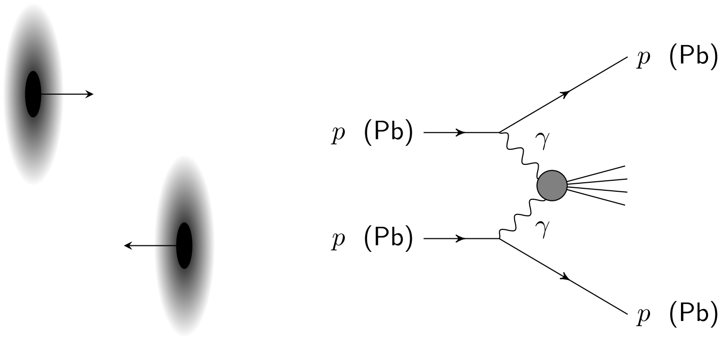
Contents

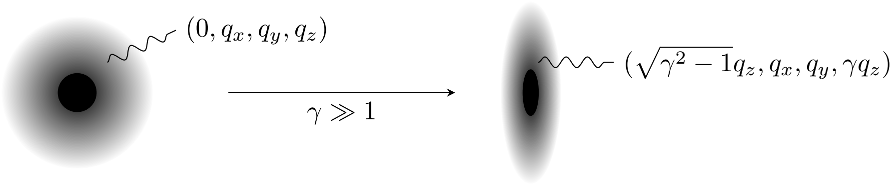
- Ultraperipheral collisions [1806.07238].
- Survival factor [2106.14842].
- Long-lived charged particles [1906.08568].
- Semi-inclusive processes [to be published].

Ultraperipheral collisions

[1806.07238]

Ultraperipheral collisions at the LHC

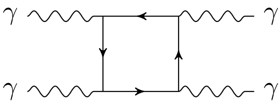




Photon virtuality: $Q^2 \equiv -q^2 = q_x^2 + q_y^2 + q_z^2 \ll (\gamma q_z)^2 \equiv \omega^2$

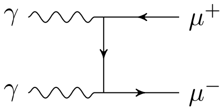
Photon-photon processes studied at LHC

$\gamma\gamma \to \gamma\gamma$



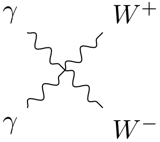
Nature Physics 13, 852 (2017)
Phys.Lett. B797, 134826 (2019)

$\gamma\gamma \to \mu^+\mu^-$



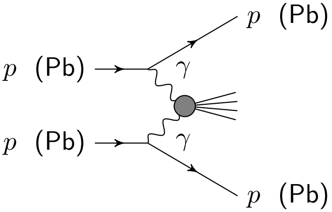
Phys.Lett. B777, 303 (2018)
JHEP 1201, 052 (2012)

$\gamma\gamma \to W^+W^-$



Phys.Rev. D94, 032011 (2016)
JHEP 1608, 119 (2016)

p vs Pb

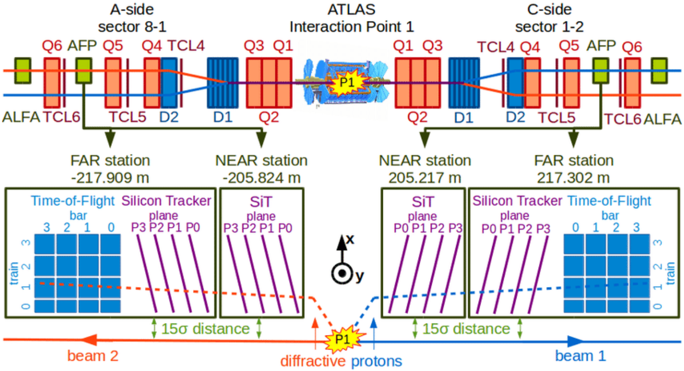


$$\sigma \sim Z^4$$

	pp	Pb Pb
Energy	13 TeV	5.02 TeV/(nucleon pair)
Z	1	82
Z^4	1	$4.5 \cdot 10^7$
Luminosity	159 fb^{-1} ratio:	2.4 nb^{-1} $6.6 \cdot 10^7$
Duration	21 months (Run 2)	2 months (2015, 2018)
$\sqrt{s_{\gamma\gamma}}$	$\lesssim 2.6$ TeV	$\lesssim 160$ GeV

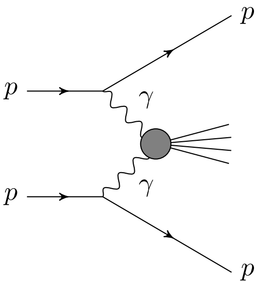
p vs Pb: forward detectors

[1909.10827]



Distance from the IP, m	200	420
ξ range	$0.015–0.15$	$0.002–0.02$
6.5 TeV p energy loss, GeV	$97.5–975$	$13–130$
in the center-of-mass frame, MeV	$14–141$	$1.9–19$
0.5 PeV ^{208}Pb energy loss, TeV	$7.8–78$	$1.0–10$
in the center-of-mass frame, GeV	$2.9–29$	$0.37–3.7$

Equivalent photons approximation



$$\sigma(pp \to ppX) = \int\limits_0^\infty \mathrm{d}\omega_1 \int\limits_0^\infty \mathrm{d}\omega_2\, \sigma(\gamma\gamma \to X)\, n_p(\omega_1)\, n_p(\omega_2)$$

$$n_p(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int\limits_0^\infty \left[\frac{F_p(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp^2 + (\omega/\gamma)^2}\right]^2 q_\perp^3 \mathrm{d}q_\perp$$

$$Q^2 \equiv -q^2 = q_\perp^2 + \left(\frac{\omega}{\gamma}\right)^2$$

Proton form factor

$$F_p(Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau}$$

$$G_E(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}$$

$$G_M(Q^2) = \frac{\mu_p}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}$$

$$\tau = \frac{Q^2}{4m_p^2} \lesssim 0.01$$

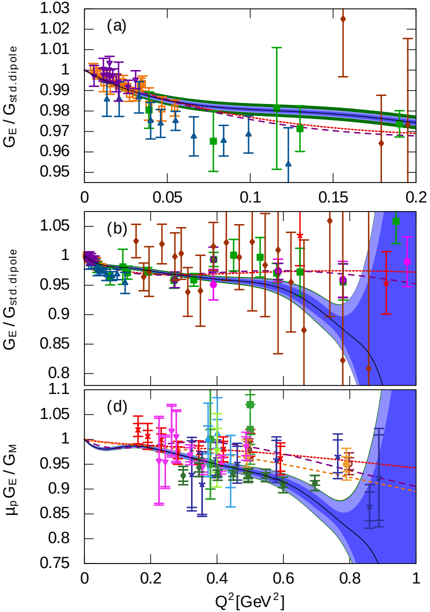
$$\mu_p = 2.79$$

$$\Lambda^2_{\text{std.}} = 0.71\ \text{GeV}^2$$

Proton radius

$r_p = 0.8414\ \text{fm} = \sqrt{12}/\Lambda$

$\Rightarrow \Lambda^2 = 0.66\ \text{GeV}^2$.



[1307.6227]

Proton EPA spectrum

Proton form factor with magnetic contribution taken into account:

$$F_p(Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau},$$

$$G_E(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}, \qquad G_M(Q^2) = \frac{\mu_p}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}$$

$$\tau = \frac{Q^2}{4m_p^2} \lesssim 0.01,\ \Lambda^2 = 0.61\ \text{GeV}^2,\ \mu_p = 2.79.$$

EPA spectrum:

$$\begin{aligned} n_p(\omega) = \frac{\alpha}{\pi\omega}\Bigg\{ & \left(1+4u-2(\mu_p-1)\frac{u}{v}\right)\ln\left(1+\frac{1}{u}\right) - \frac{24u^2+42u+17}{6(u+1)^2} \\ & + \frac{\mu_p-1}{(v-1)^4}\left[\frac{\mu_p-1}{v-1}(1+4u+3v) - 2\left(1+\frac{u}{v}\right)\right]\ln\frac{u+v}{u+1} \\ & + (\mu_p-1)\,\frac{6u^2(v^2-3v+3)+3u(3v^2-9v+10)+2v^2-7v+11}{3(u+1)^2(v-1)^3} \\ & - (\mu_p-1)^2\,\frac{24u^2+6u(v+7)-v^2+8v+17}{6(u+1)^2(v-1)^4}\Bigg\}, \end{aligned}$$

$$u = (\omega/\Lambda\gamma)^2,\ v = (2m_p/\Lambda)^2.$$

^{208}Pb form factor

Charge distribution is provided in form of Bessel spherical functions:

$$\rho(r) = \sum_{k=1}^{N} a_k j_0(k\pi r/R)\Theta(R-r);\ j_0(x) = \sin(x)/x.$$

a_k, R are numerical parameters.
Form factor is the Fourier transform of charge:

$$F_{\mathsf{Pb}}(Q^2) = \frac{\int \rho(r) e^{i\vec{q}\vec{r}} d^3 r}{\int \rho(r) d^3 r} = \frac{\sin QR}{QR} \frac{\sum_{k=1}^{N} \frac{(-1)^k a_k}{(k\pi)^2 - (QR)^2}}{\sum_{k=1}^{N} \frac{(-1)^k a_k}{(k\pi)^2}}.$$

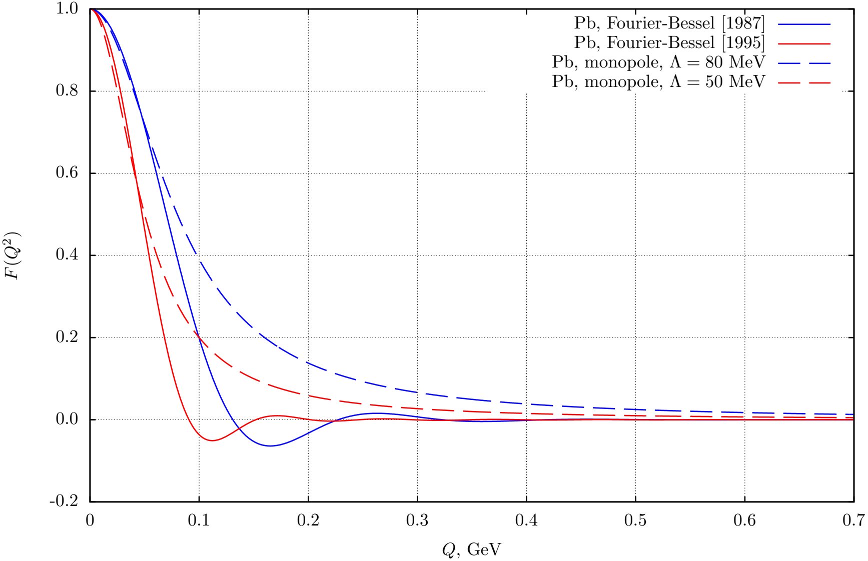
EPA spectrum:

$$n_{\mathsf{Pb}}(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int_0^\infty \left[\frac{F(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp^2 + (\omega/\gamma)^2}\right]^2 q_\perp^3 \, dq_\perp$$

Monopole approximation:

$$F_1(Q^2) = \frac{1}{1 + Q^2/\Lambda^2},\ n_1(\omega) = \frac{Z^2\alpha}{\pi\omega}\left[(1+2u)\ln\left(1 + \frac{1}{u}\right) - 2\right],\ u = \left(\frac{\omega}{\Lambda\gamma}\right)^2$$

^{208}Pb form factor



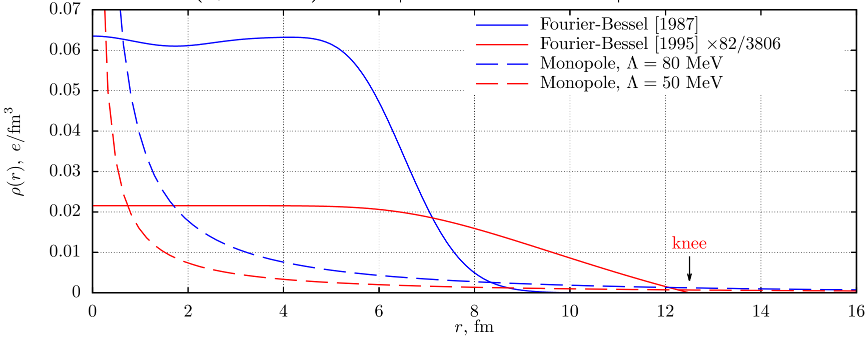
[1987]: At.Data and Nucl.Data Tabl. 36, 495 (1987)

[1995]: At.Data and Nucl.Data Tabl. 60, 177 (1995)

^{208}Pb charge density

$$\rho(r) = \sum_{k=1}^{N} a_k\, j_0(k\pi r/R)\,\Theta(R-r), \quad F(Q) = \frac{\int \rho(r) e^{i\vec{q}\vec{r}} d^3 r}{\int \rho(r) d^3 r}$$

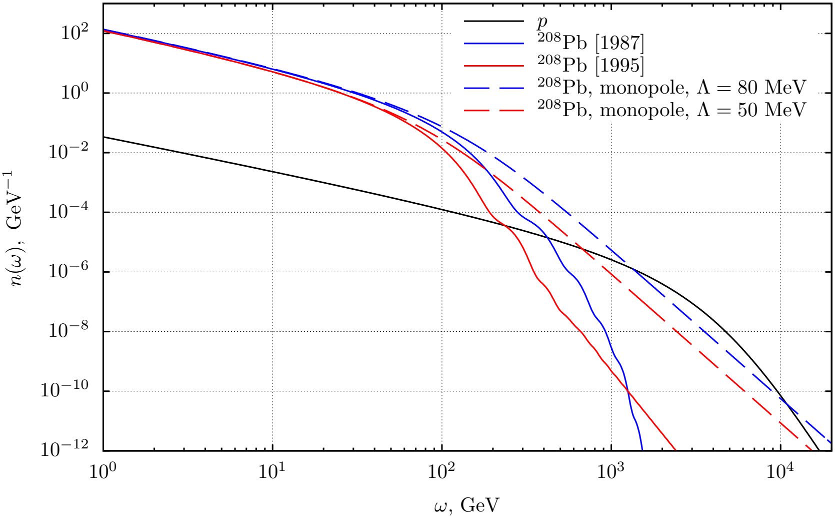
	[1987] Quoted	[1987] Actual	[1995] Quoted	[1995] Actual
$Z = \int \rho(r) d^3 r$	82	82.03	82	3806
$\langle r^2 \rangle^{1/2} = \left(\frac{\int \rho(r) r^2 d^3 r}{\int \rho(r) d^3 r} \right)^{\frac{1}{2}}$, fm	5.499(1)	5.501	5.4785	8.0850



[1987]: At.Data and Nucl.Data Tabl. 36, 495 (1987)
[1995]: At.Data and Nucl.Data Tabl. 60, 177 (1995) ?

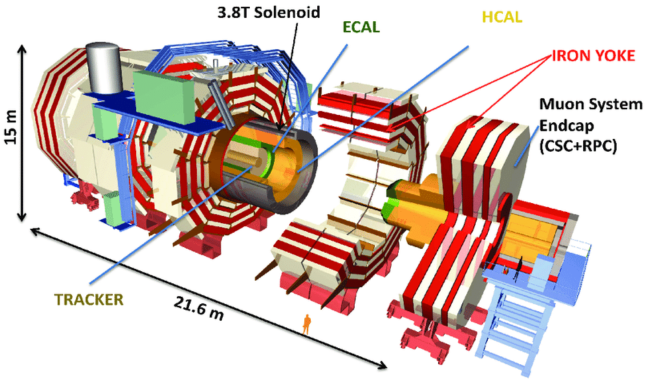
EPA spectra

$E_p = 6.5$ TeV, $E_{\mathrm{Pb}} = 522$ TeV $(2.5$ TeV/nucleon$)$



$\hat{q}_p \gamma_p = 1.3$ TeV, $\hat{q}_{\mathrm{Pb}} \gamma_{\mathrm{Pb}} = 79$ GeV

CMS detector

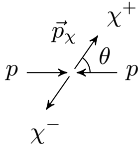


Typical experimental requirements: $|\eta| < \hat{\eta}$, $p_T > \hat{p}_T$.

$\eta = -\ln \tan \frac{\theta}{2}$,

$y = \frac{1}{2} \ln \frac{E_\chi + p_\chi \cos\theta}{E_\chi - p_\chi \cos\theta} \overset{m_\chi \ll E_\chi}{=} \eta$

$p_T = p_\chi \sin\theta$



Fiducial cross section

$$\frac{\mathrm{d}\sigma_{\mathsf{fid}}(pp \to pp\chi^+\chi^-)}{\mathrm{d}s} = \int\limits_{\max\left(\hat{p}_T,\, \frac{\sqrt{s}}{2\cosh\hat{\eta}}\sqrt{1-\frac{4m_\chi^2}{s}}\right)}^{\frac{\sqrt{s}}{2}\sqrt{1-\frac{4m_\chi^2}{s}}} \mathrm{d}p_T \, \frac{\mathrm{d}\sigma(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T} \, \frac{\mathrm{d}\hat{L}}{\mathrm{d}s}$$

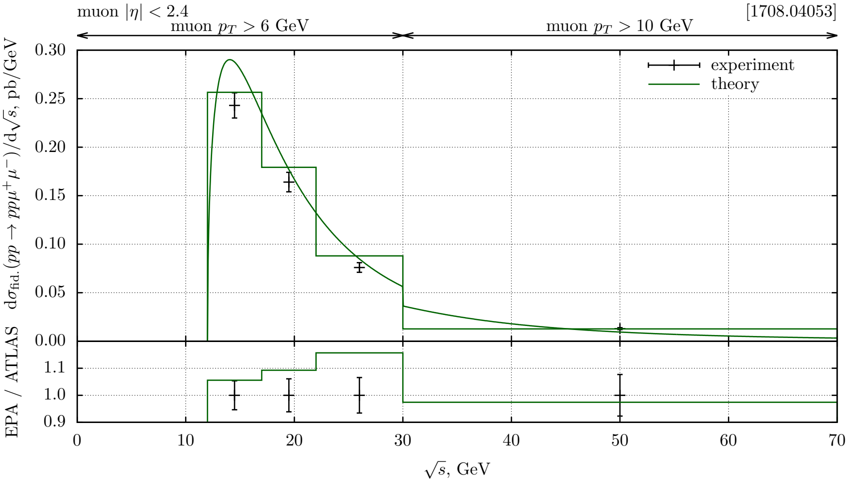
Photon-photon luminosity:

$$\frac{\mathrm{d}\hat{L}}{\mathrm{d}s} = \frac{1}{4} \int\limits_{-\hat{y}}^{\hat{y}} \mathrm{d}y \, n\left(\frac{\sqrt{s}}{2}\mathrm{e}^{y}\right) \, n\left(\frac{\sqrt{s}}{2}\mathrm{e}^{-y}\right),$$

$\sqrt{s}$ — invariant mass of $\chi^+\chi^-$ pair, $y = \frac{1}{2}\ln\frac{\omega_1}{\omega_2}$ — its rapidity,

$$\hat{y} = \operatorname{asinh}\left[\frac{\sqrt{s}p_T}{2(p_T^2 + m_\chi^2)}\left(\sinh\hat{\eta} - \sqrt{\cosh^2\hat{\eta} + \frac{m_\chi^2}{p_T^2}} \cdot \sqrt{1 - \frac{4(p_T^2 + m_\chi^2)}{s}}\right)\right]$$

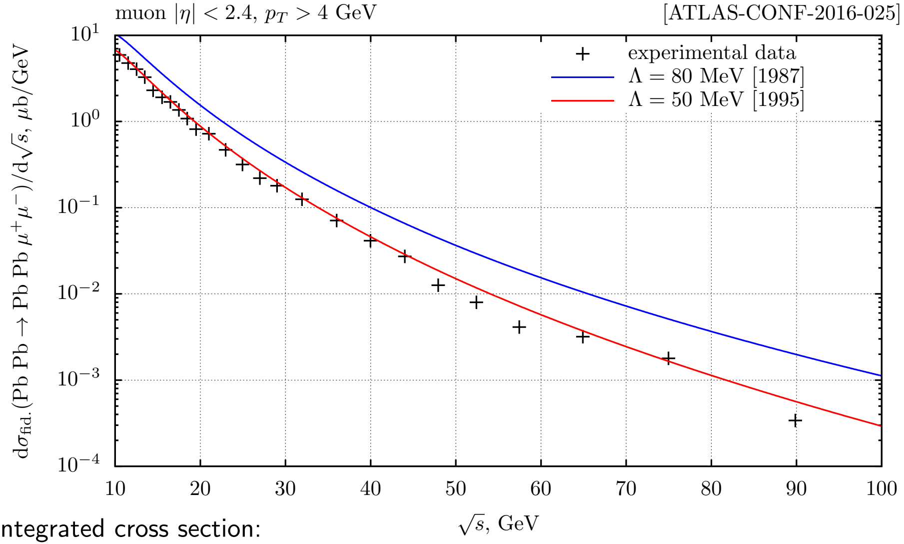
ATLAS experiment: $pp \to pp\mu^+\mu^-$



Integrated cross section:

- Experiment: 3.12 ± 0.07 (stat.) $\pm\, 0.10$ (syst.) pb.
- Theory: 3.39 pb.

ATLAS experiment: Pb Pb $\to$ Pb Pb $\mu^+\mu^-$ (preliminary)



Integrated cross section:

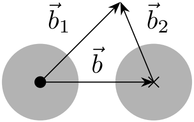
- Experiment: 32.3 ± 0.3 (stat.)$^{+4.0}_{-3.4}$ (syst.) μb.
- Theory: 34.4 μb ($\Lambda = 50$ MeV) or 57.4 μb ($\Lambda = 80$ MeV).

Survival factor

[2106.14842]

Correction from strong interactions

Assuming only electromagnetic interactions:



$$n(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int\limits_0^\infty \left[\frac{F(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp^2 + (\omega/\gamma)^2} \right]^2 q_\perp^3 \mathrm{d}q_\perp ,$$

$$\sigma(pp \to ppX) = \int\limits_0^\infty \mathrm{d}\omega_1 \int\limits_0^\infty \mathrm{d}\omega_2 \, \sigma(\gamma\gamma \to X) \, n_p(\omega_1) \, n_p(\omega_2).$$

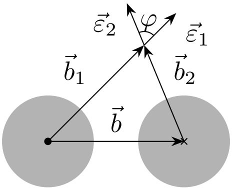
Including strong interactions:

$$n(\omega) = \int n(b,\omega) \, \mathrm{d}^2 b, \quad n(b,\omega) = \frac{Z^2\alpha}{\pi^2\omega} \left[\int\limits_0^\infty \frac{F(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp^2 + (\omega/\gamma)^2} J_1(bq_\perp) \, q_\perp^2 \mathrm{d}q_\perp \right]^2 ,$$

$$\sigma(pp \to ppX) \\ = \int\limits_0^\infty \mathrm{d}\omega_1 \int\limits_0^\infty \mathrm{d}\omega_2 \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, \sigma(\gamma\gamma \to X) \, n_p(b_1,\omega_1) \, n_p(b_2,\omega_2) \, P(|\vec{b}_1 - \vec{b}_2|).$$

$P(b)$ is the probability for the protons to survive after the collision with the impact parameter b.

Polarization



$$\sigma(pp \to ppX) = \int_0^\infty \mathrm{d}s \left[\sigma_\parallel(\gamma\gamma \to X) \frac{\mathrm{d}L_\parallel}{\mathrm{d}s} + \sigma_\perp(\gamma\gamma \to X) \frac{\mathrm{d}L_\perp}{\mathrm{d}s} \right],$$

where

$$\frac{\mathrm{d}L_\parallel}{\mathrm{d}s} = \frac{1}{4} \int_{-\infty}^{\infty} \mathrm{d}y \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right) n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right) P(b) \cos^2\varphi,$$

$$\frac{\mathrm{d}L_\perp}{\mathrm{d}s} = \frac{1}{4} \int_{-\infty}^{\infty} \mathrm{d}y \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right) n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right) P(b) \sin^2\varphi$$

are photon-photon luminosities.

Survival factor

$$\sigma(pp \to ppX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, \sigma(\gamma\gamma \to X) \, n_p(b_1, \omega_1) \, n_p(b_2, \omega_2) \, P(|\vec{b}_1 - \vec{b}_2|)$$

vs

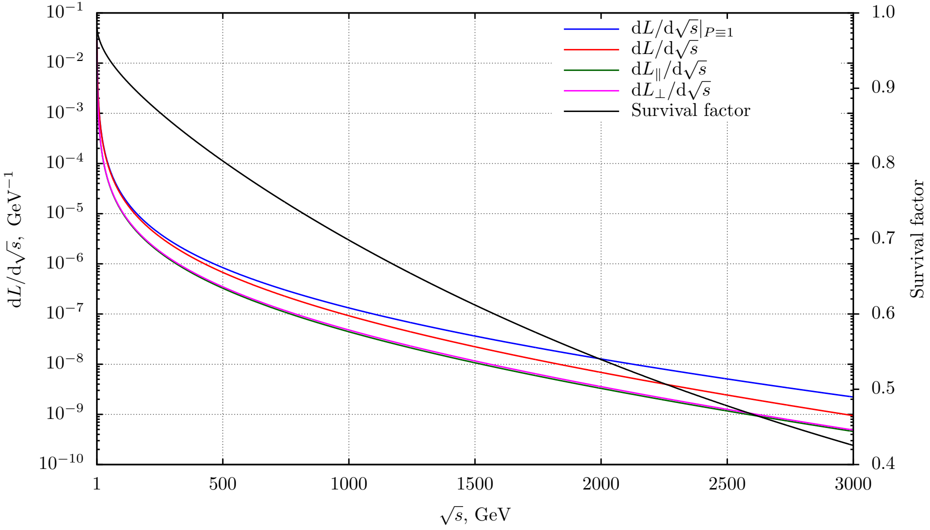
$$\sigma(pp \to ppX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \, \sigma(\gamma\gamma \to X) \, n_p(\omega_1) \, n_p(\omega_2)$$

Neglecting the polarization,

$$S(s) = \frac{\mathrm{d}L/\mathrm{d}s}{\mathrm{d}L/\mathrm{d}s|_{P=1}},$$

where $L = L_\parallel + L_\perp$.

Survival factor in pp collisions



Survival factor in Pb Pb collisions — no result

$$F_{\mathsf{Pb}}(Q^2) = \frac{\sin QR}{QR} \frac{\sum\limits_{k=1}^{N} \frac{(-1)^k a_k}{(k\pi)^2 - (QR)^2}}{\sum\limits_{k=1}^{N} \frac{(-1)^k a_k}{(k\pi)^2}}$$

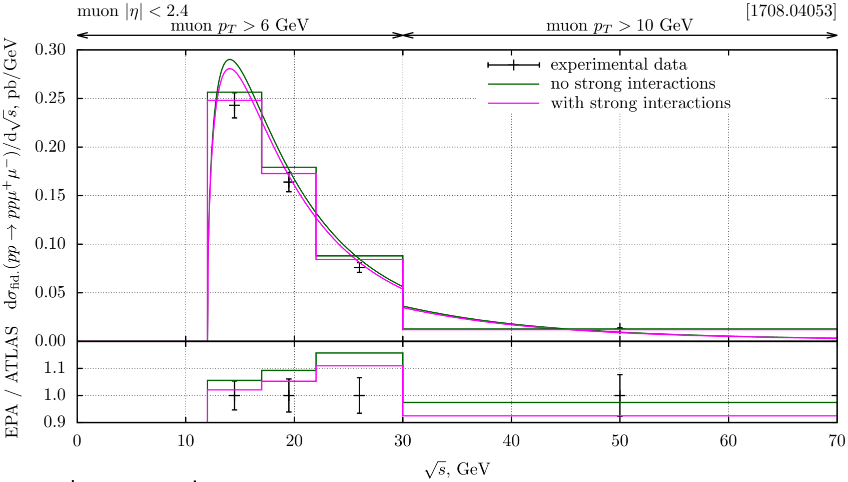
$$n(b,\omega) = \frac{Z^2\alpha}{\pi^2\omega} \left[\int\limits_0^\infty \frac{F(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp^2 + (\omega/\gamma)^2} J_1(bq_\perp) q_\perp^2 \, \mathrm{d}q_\perp \right]^2$$

$$= \frac{Z^2\alpha}{\pi^2\omega R^4} \left[\sum_{k=1}^{K} \frac{(-1)^k a_k}{(k\pi)^2} \right]^{-2} \times$$

$$\left[\sum_{k=1}^{K} (-1)^k a_k \int\limits_0^\infty \frac{x^2 \sin\sqrt{x^2+y^2}\, J_1(cx)}{(x^2+y^2-(k\pi)^2)(x^2+y^2)^{\frac{3}{2}}} \, \mathrm{d}x \right]^2$$

$$x = q_\perp R, \; y = R\omega/\gamma, \; c = b/R.$$

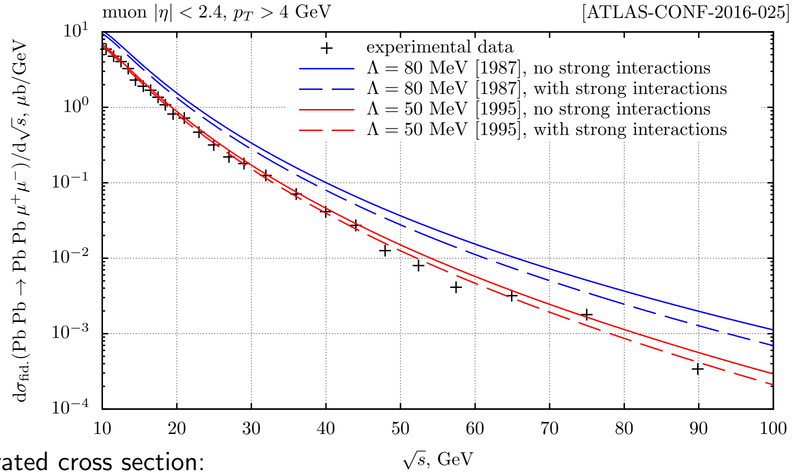
ATLAS experiment: $pp \to pp\mu^+\mu^-$



Integrated cross section:

- Experiment: 3.12 ± 0.07 (stat.) $\pm\, 0.10$ (syst.) pb.
- No strong interactions: 3.39 pb.
- With strong interactions: 3.26 pb.

ATLAS experiment: Pb Pb $\to$ Pb Pb $\mu^+\mu^-$ (preliminary)

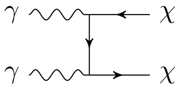


Integrated cross section:

Λ	50 MeV	80 MeV
No strong interactions	$34.4\ \mu$b	$57.4\ \mu$b
With strong interactions	$32.3\ \mu$b	$50.8\ \mu$b
Experiment	32.3 ± 0.3 (stat.)$^{+4.0}_{-3.4}$ (syst.) μb.	

Long-lived charged particles

[1906.08568]

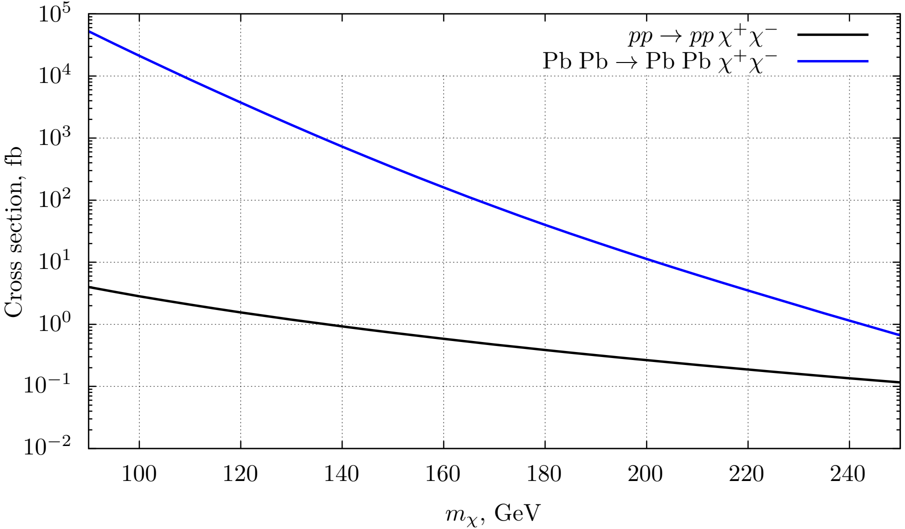


- Long-lived charged particles
  - Live long enough to escape the detector (like muons).
  - Usual search techniques: $\mathrm{d}E/\mathrm{d}x$, time of flight.
  - Current bounds [1506.09173, 1609.08382, 1902.01636] are model-dependent.
  - Example: SUSY chargino nearly degenerated with neutralino, $m_\chi \gtrsim 100$ GeV.
- UPC approach:
  - The particles leave tracks in the central detector allowing for reconstruction of their momenta $\vec{p}_1$, $\vec{p}_2$.
  - Forward detectors provide the proton energies after the collision E_1, E_2.
  - Collision kinematics is reconstructed. The mass of the particle

$$m = \sqrt{\frac{(2E_1E_2 + \vec{p}_1\vec{p}_2)^2 - \vec{p}_1^{\,2}\vec{p}_2^{\,2}}{4E_1E_2 + (\vec{p}_1 + \vec{p}_2)^2}}.$$

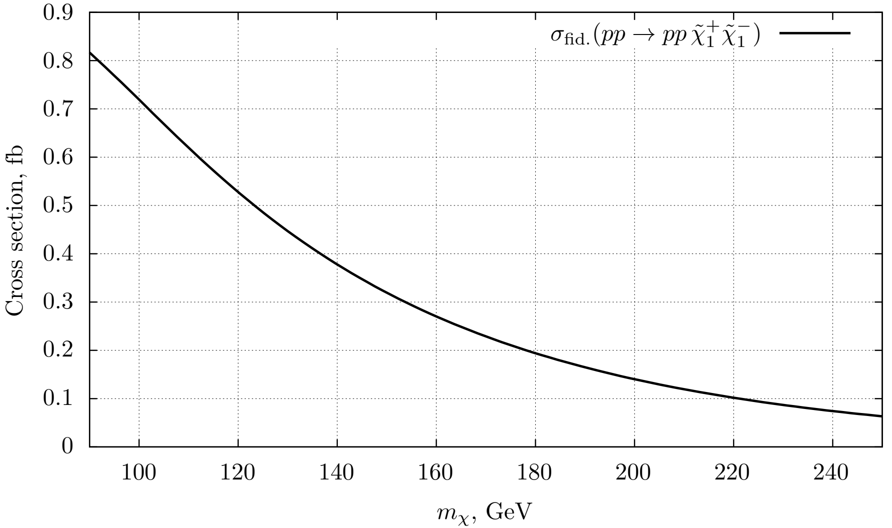
- Complementary to $\mathrm{d}E/\mathrm{d}x$ or time of flight measurements.
- Background:
  - $pp \to pp\mu^+\mu^-$ (and other processes producing muons).
  - Pileup and diffractive scattering.

Fermion pair production



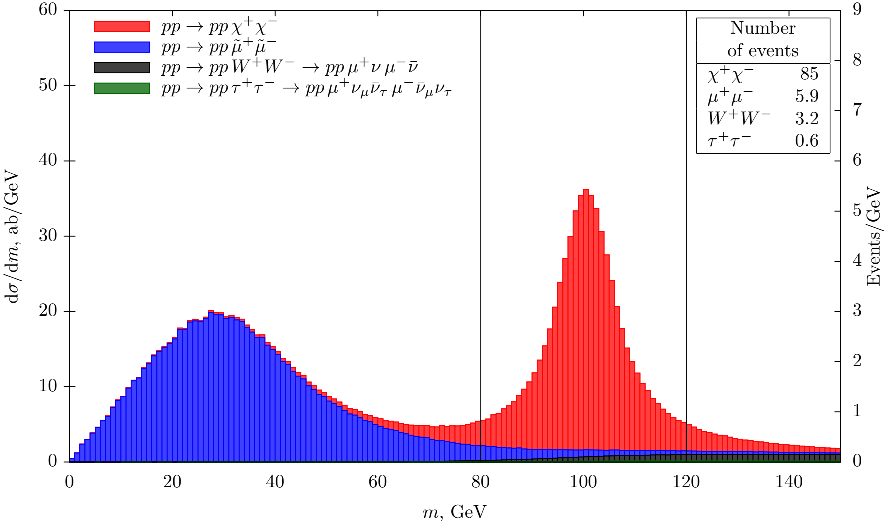
Available integrated luminosity: pp — 159 fb^{-1}, Pb Pb — 2.4 nb^{-1}.

Fermion pair production (fiducial)



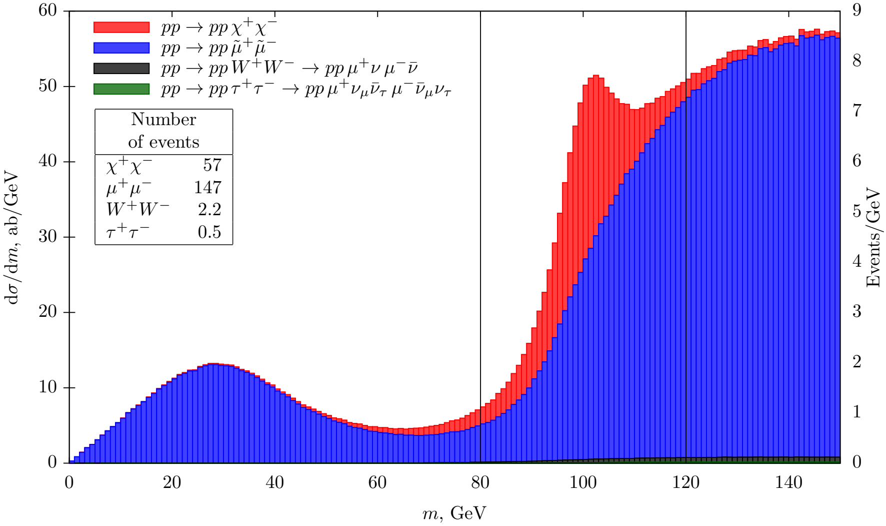
$p_T > 20$ GeV, $|\eta| < 2.5$, both protons hit the forward detectors.

Expected signal (no pileup)



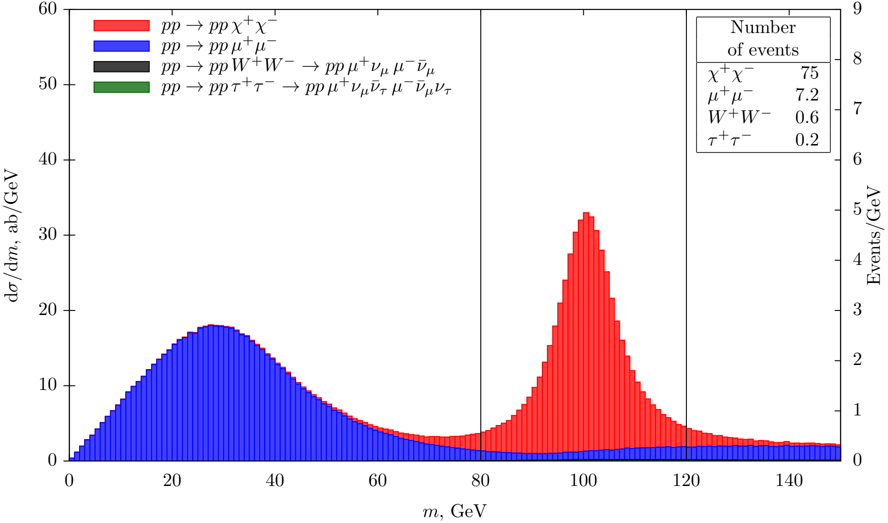
Assuming $m_\chi = 100$ GeV and integrated luminosity 150/fb.

Expected signal (with pileup)



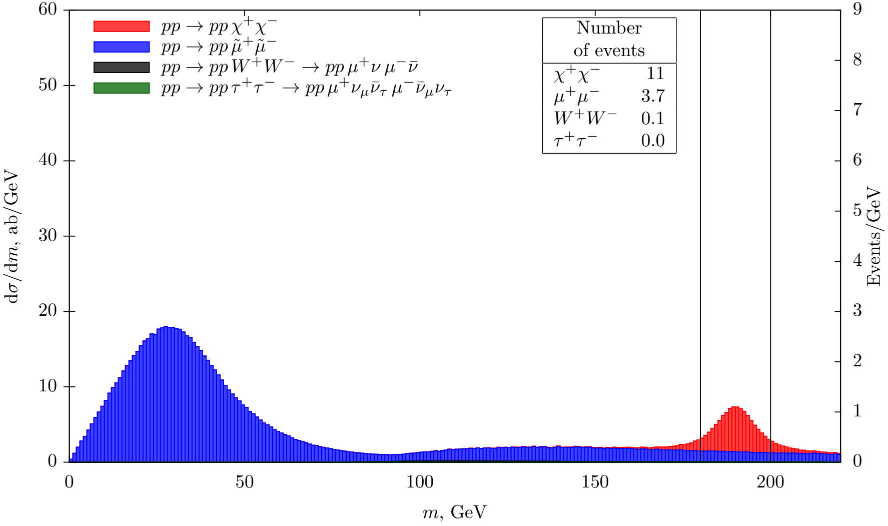
Assuming pileup of 50 collisions at once.

Expected signal (with pileup and longitudinal selection)

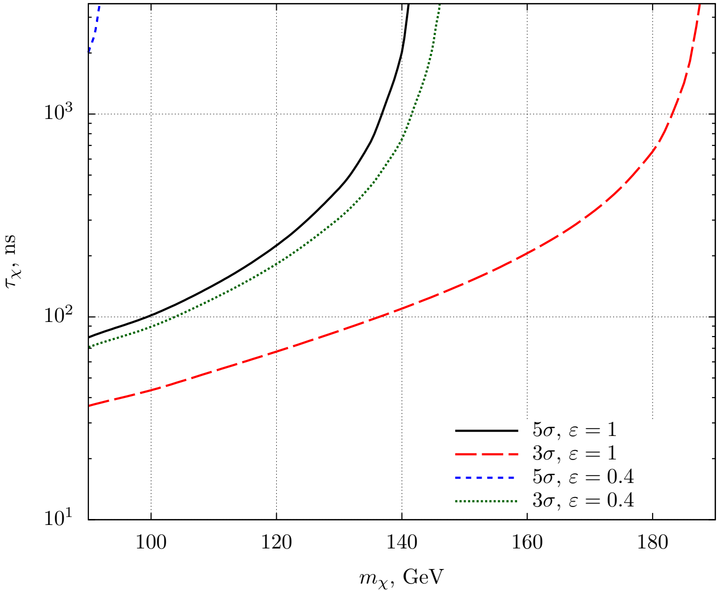


Longitudinal selection: $|p_{\parallel,1} + p_{\parallel,2} - (E_1 - E_2)| < 20$ GeV.

Expected signal (with pileup and longitudinal selection)



$m_\chi = 190$ GeV.

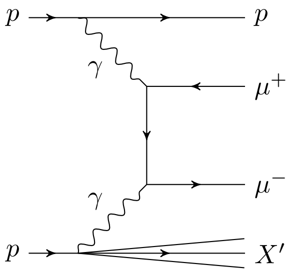


Assuming integrated luminosity 150/fb.

Semi-inclusive processes

[to be published]

[2009.14537] measurement



Experimental selections:

- $p_T > 15$ GeV.
- $|\eta| < 2.4$.
- $p_T^{\mu\mu} < 5$ GeV.
- 20 GeV $< m_{\mu\mu} <$ 70 GeV or $m_{\mu\mu} > 105$ GeV.
- At least one proton hits a forward detector.

$$\sigma(pp \to pX\mu^+\mu^-) = \sum_q \sigma(pq \to pq\mu^+\mu^-),$$

For $\left(\frac{p_T^{\mu\mu}}{m_{\mu\mu}}\right)^2 \ll 1, \sigma(pq \to pq\mu^+\mu^-) = \int\limits_0^\infty \mathrm{d}\omega_1 \int\limits_0^\infty \mathrm{d}\omega_2\, \sigma(\gamma\gamma \to \mu^+\mu^-)\, n_p(\omega_1)\, n_q(\omega_2)$

$$n_q(\omega) = \frac{\alpha}{\pi\omega} \int\limits_{\omega/E}^{1} \mathrm{d}x \int\limits_{(\omega/x\gamma)^2}^{(p_T^{\mu\mu})^2} \mathrm{d}Q^2\, \frac{q_\perp^2}{Q^2} f_q(x, Q^2)$$

Preliminary results

- Experiment: 7.2 ± 1.6 (stat.) $\pm\, 0.9$ (syst.) $\pm\, 0.2$ (lumi.) fb.
- Exclusive process ($pp \to pp\mu^+\mu^-$): 8.0 fb.
- Inclusive process ($pp \to pX\mu^+\mu^-$): 8.7 fb.

Survival factor should reduce the calculated values by $\sim 10\%$. Errors are to be estimated yet.

Conclusions

- Ultraperipheral collisions are a clean source of photon-photon collisions.
- Proton dipole form factor yields accurate results. Lead form factor is weird.
- Survival factor gives corrections of a few percents in the measurements considered.
- A new technique for detecting long-lived charged particles with the help of forward detectors. With 150/fb of integrated luminosity, particles with lifetimes down to $\sim 10^2$ ns and masses up to ~ 150 GeV can be detected.
- Our calculation of semi-inclusive muon pair production cross section overshoots the measurement by a factor of 2.
- `libepa` (https://github.com/jini-zh/libepa) — A library for calculations of cross sections of ultraperipheral collisions under the equivalent photons approximation.

Backup

Proton EPA spectrum

$$F_p(Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau},$$

$$G_E(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}, \quad G_M(Q^2) = \frac{\mu_p}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2}, \quad \tau = -\frac{Q^2}{4m_p^2},$$

$$n_p(b,\omega) = \frac{\alpha}{\pi^2\omega}\left[\int_0^\infty \frac{F_p(q_\perp^2 + (\omega/\gamma)^2)}{q_\perp^2 + (\omega/\gamma)^2} J_1(bq_\perp) q_\perp^2 \mathrm{d}q_\perp\right]^2$$

$$= \frac{\alpha}{\pi^2\omega}\left[\frac{\omega}{\gamma}K_1\left(\frac{b\omega}{\gamma}\right) - \left(1 + \frac{(\mu_p - 1)\frac{\Lambda^4}{16m_p^4}}{\left(1-\frac{\Lambda^2}{4m_p^2}\right)^2}\right)\sqrt{\Lambda^2 + \frac{\omega^2}{\gamma^2}}\, K_1\left(b\sqrt{\Lambda^2+\frac{\omega^2}{\gamma^2}}\right)\right.$$

$$+ \frac{(\mu_p - 1)\frac{\Lambda^4}{16m_p^4}}{\left(1-\frac{\Lambda^2}{4m_p^2}\right)^2}\sqrt{4m_p^2 + \frac{\omega^2}{\gamma^2}}\, K_1\left(b\sqrt{4m_p^2+\frac{\omega^2}{\gamma^2}}\right)$$

$$\left. - \frac{1-\frac{\mu_p\Lambda^2}{4m_p^2}}{1-\frac{\Lambda^2}{4m_p^2}} \cdot \frac{b\Lambda^2}{2} K_0\left(b\sqrt{\Lambda^2+\frac{\omega^2}{\gamma^2}}\right)\right]^2,$$

$\gamma\gamma$ luminosities in pp collisions

The probability for both protons to survive [hep-ph/0608271],

$$P(b) = \left(1 - \mathrm{e}^{-\frac{b^2}{2B}}\right)^2, \ B = 21 \text{ GeV}^2.$$

Photon-photon luminosities:

$$\frac{\mathrm{d}L_{pp}^{\parallel}}{\mathrm{d}s} = \frac{\pi^2}{2} \int\limits_{\hat{r}}^{\infty} b_1 \, \mathrm{d}b_1 \int\limits_{\hat{r}}^{\infty} b_2 \, \mathrm{d}b_2 \int\limits_{-\infty}^{\infty} \mathrm{d}y \, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right) n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right)$$

$$\times \left\{1 - 2\mathrm{e}^{-\frac{b_1^2+b_2^2}{2B}} \left[I_0\left(\frac{b_1 b_2}{B}\right) + I_2\left(\frac{b_1 b_2}{B}\right)\right] + \mathrm{e}^{-\frac{b_1^2+b_2^2}{B}} \left[I_0\left(\frac{2b_1 b_2}{B}\right) + I_2\left(\frac{2b_1 b_2}{B}\right)\right]\right\},$$

$$\frac{\mathrm{d}L_{pp}^{\perp}}{\mathrm{d}s} = \frac{\pi^2}{2} \int\limits_{\hat{r}}^{\infty} b_1 \, \mathrm{d}b_1 \int\limits_{\hat{r}}^{\infty} b_2 \, \mathrm{d}b_2 \int\limits_{-\infty}^{\infty} \mathrm{d}y \, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right) n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right)$$

$$\times \left\{1 - 2\mathrm{e}^{-\frac{b_1^2+b_2^2}{2B}} \left[I_0\left(\frac{b_1 b_2}{B}\right) - I_2\left(\frac{b_1 b_2}{B}\right)\right] + \mathrm{e}^{-\frac{b_1^2+b_2^2}{B}} \left[I_0\left(\frac{2b_1 b_2}{B}\right) - I_2\left(\frac{2b_1 b_2}{B}\right)\right]\right\}.$$

Fiducial cross section

$$\frac{\mathrm{d}\sigma_{\text{fid.}}(pp \to pp\chi^+\chi^-)}{\mathrm{d}s} = \int\limits_{\max\left(\hat{p}_T,\, \frac{\sqrt{s}}{2\cosh\hat{\eta}}\sqrt{1-\frac{4m_\chi^2}{s}}\right)}^{\frac{\sqrt{s}}{2}\sqrt{1-\frac{4m_\chi^2}{s}}} \mathrm{d}p_T \left[\frac{\mathrm{d}\sigma_\parallel(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T}\frac{\mathrm{d}\hat{L}^\parallel}{\mathrm{d}s} + \frac{\mathrm{d}\sigma_\perp(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T}\frac{\mathrm{d}\hat{L}^\perp}{\mathrm{d}s}\right]$$

Differential photon-photon cross sections:

$$\frac{\mathrm{d}\sigma_\parallel(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T} = \frac{8\pi\alpha^2 p_T}{s(p_T^2+m_\chi^2)} \cdot \frac{1-\dfrac{2(p_T^4+2m_\chi^4)}{s(p_T^2+m_\chi^2)}}{\sqrt{1-\dfrac{4(p_T^2+m_\chi^2)}{s}}}$$

$$\frac{\mathrm{d}\sigma_\perp(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T} = \frac{8\pi\alpha^2 p_T}{s(p_T^2+m_\chi^2)} \cdot \frac{1-\dfrac{2p_T^4}{s(p_T^2+m_\chi^2)}}{\sqrt{1-\dfrac{4(p_T^2+m_\chi^2)}{s}}}$$

Fiducial cross section

Fiducial luminosity:

$$\frac{\mathrm{d}\hat{L}_{\parallel}}{\mathrm{d}s} = \frac{1}{4} \iint \mathrm{d}^2 b_1 \, \mathrm{d}^2 b_2 \, P(b) \int_{-\hat{y}}^{\hat{y}} \mathrm{d}y \, n\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right) \, n\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right) \, \cos^2\varphi$$

$$\frac{\mathrm{d}\hat{L}_{\perp}}{\mathrm{d}s} = \frac{1}{4} \iint \mathrm{d}^2 b_1 \, \mathrm{d}^2 b_2 \, P(b) \int_{-\hat{y}}^{\hat{y}} \mathrm{d}y \, n\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right) \, n\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right) \, \sin^2\varphi$$

Rapidity cut:

$$\hat{y} = \operatorname{asinh}\left[\frac{\sqrt{s}p_T}{2(p_T^2 + m_\chi^2)}\left(\sinh\hat{\eta} - \sqrt{\cosh^2\hat{\eta} + \frac{m_\chi^2}{p_T^2}} \cdot \sqrt{1 - \frac{4(p_T^2 + m_\chi^2)}{s}}\right)\right]$$